Optimal or Historical? The Design and Evolution of Rail Transport Networks in Australia.

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Abstract
Australia's transport network has evolved organically over the past 200 years in response to geographic conditions and economic demands. What would the rail network look like if it were designed with today's demands and constraints. The paper uses some optimal network modelling with minimum spanning trees and Steiner trees to create a structure which is compared with the historical networks.

Introduction

Australia's transport network reflects the growth and development of the continent since its colonisation in the 18th century. The country first developed as a series of isolated ports that were dependent upon their agricultural hinterlands for support and supplies from England. The roads in the new settled country largely followed the routes of the explorers that went in search of richer agricultural lands[1] Some of the first roads in New South Wales were in fact toll roads. Rail began in the cities in Australia in the middle of the 19th century initially as a means of urban transport. Most railways began as private ventures but in response to volatile economic conditions they failed and each were taken over by state governments, because of the essential nature of rail for the growth of each state. In response to the growing rural economy and the discovery of gold in each State, the rail networks expanded rapidly in the latter half of the 19th century. The rail network and the road networks both had their focus on the main port cities which had become the capital cities of each State. The independence of each of the cities and their states can be seen in the different rail gauges that grew outwards from each capital city.

This paper examines the rail network that has evolved and asks the question, what would the rail network look like if it were in fact designed with today's current demands? What should the rail network look like if it were designed in some optimal manner[2]? In order to answer this question, some basic network theory is used to develop what is known as a minimum spanning tree for the rail network. The idea of a scale free network is also explored as a means of describing the rail network[3].

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[1] History of Roads in Australia, 1974 Commonwealth Year Book.
[2] Black (2003) discusses characteristics of optimal transport networks and also provides a comprehensive review of the geography of transport networks.
[3] Watts(2003) provides a straightforward review of this concept in his book "Six Degrees:"

Networks, Graphs and Scales.

A transport network is essentially a graph in mathematical terms. That is, it has vertices (nodes or terminals) and edges (links or routes of transport modes rail, road, air, sea). So a graph G can be summarised by G(V,E) where V is the number of vertices and E is the number of edges. As an example, take the South Australian rail network in figure 1 below.

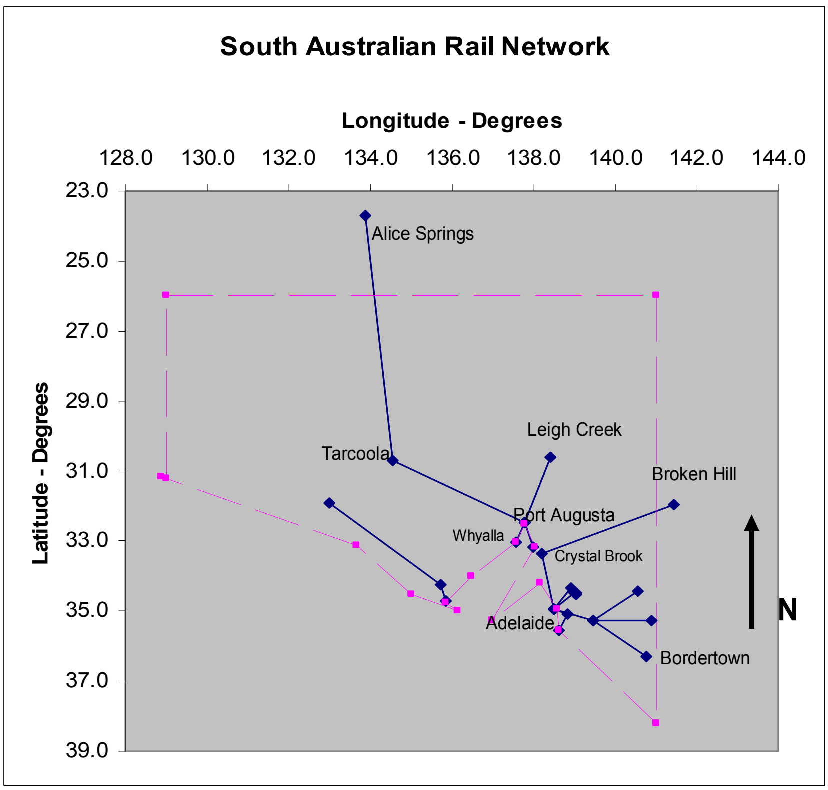

Figure 1: South Australian Rail Network

This network has 22 nodes (vertices) and coincidentally 22 links (edges) so G(22,22). There are, in this graph, potentially 231 links if every node was connected to every other node ([N x (N-1)]/2) but as can been seen from the network diagram (Figure 1) the network has relatively low connectivity. In fact, in terms of connecting all the nodes in the shortest possible manner, this network is very effective.

One of the important algorithms in graph theory is the creation of networks with the property that have a minimal length of links but with the property that all nodes still remain connected to the network. This is known as a minimal spanning tree. There are many algorithms for creating a minimal spanning tree but the simplest uses an approach known as the "greedy algorithm". Essentially this means choosing a node that is closest to an existing node and making that a link in the spanning tree. The algorithm proceeds by adding the next closest link in the network to the existing minimal spanning network. It is sometimes known as Prim's algorithm (Berry et.al. 1990). The use of the term "tree" arises because there are no circuits or loops in a minimal spanning tree. It can be shown that this simple algorithm does indeed produce the minimum connecting set of links for a network. In that sense it produces an optimal outcome in terms of minimising the length of the network – a rather valuable property given the cost of building rail lines.

The number of connections a node has with other nodes is defined as the degree of the node. Most nodes in this network have single links (Tarcoola – Port Augusta) but some nodes have more links such as Port Augusta or Adelaide. The relationship between the number of nodes and their degree has been observed by numerous studies to follow a power distribution with most nodes having a few links and a few nodes having many links.

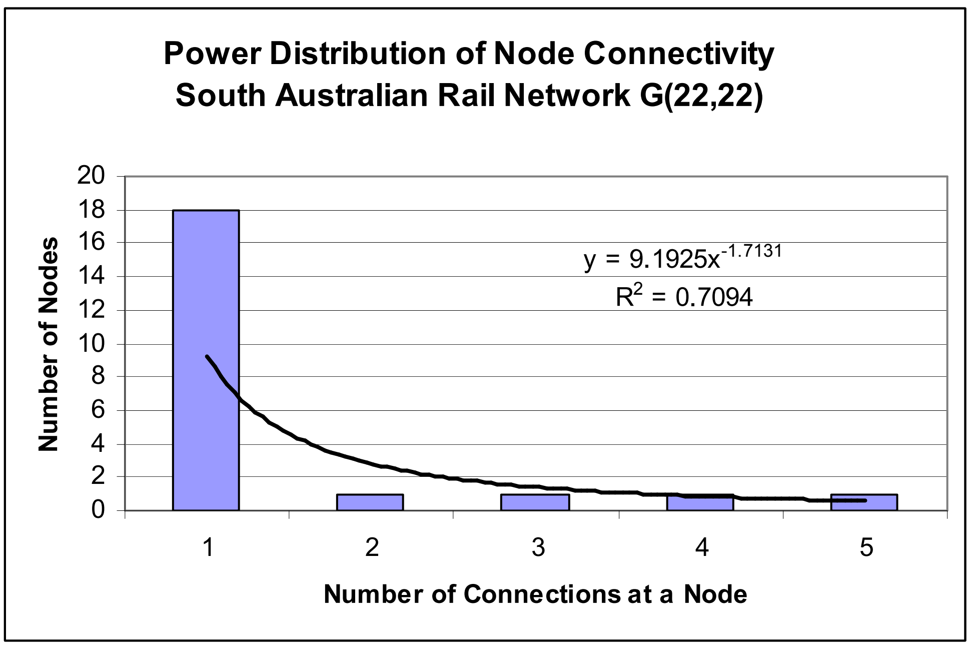

Figure 2: Nodal Connectivity of South Australian Rail Network

Networks that can be described with equations of the form $N(k) = k^{\gamma}$ are described as "scale free". This was observed by Barbasi (2003,2004) in his study of the internet and its connectivity. He expected that there would be no hierarchy on the internet so that each node would have an equal chance of directly connecting to another node. However, he found that there were a few nodes with massive connectivity, the internet hubs.

Figure 2 shows that most nodes in the South Australian rail network have a single link and a few nodes have greater connectivity. In fact Adelaide connects with five other nodes, Port Augusta four and Tailem Bend three. Adelaide, is the hub of the South Australian network as one would expect given its historical significance.

Scale free networks are so termed because the structure of the network remains the same no matter how closely one examines the network. It has "fractal" properties in that there is self similarity independent of the scale of observation of the network. In the next section, when we examine the Australian rail network we will observe the same structure – that a few nodes are well connected whilst most others remain weakly connected.

This property is important because it can be shown that this means the network is quite robust to disruptions. If some links are broken, for what ever reason, a high degree of connectivity remains in the network. Many real world networks have this property (telephones, airlines and the internet to name but a few).

Optimal Rail Networks

A complete graph of the Australian network with all of nodes would require a table with over a thousand nodes and the resulting connectivity matrix would have well over a million cells. In this paper, a sketch planning network was created that uses only the major nodes associated with the network. This was created using the source "Australian Rail Maps"[4] Figure 3 shows the resulting network for Australia.

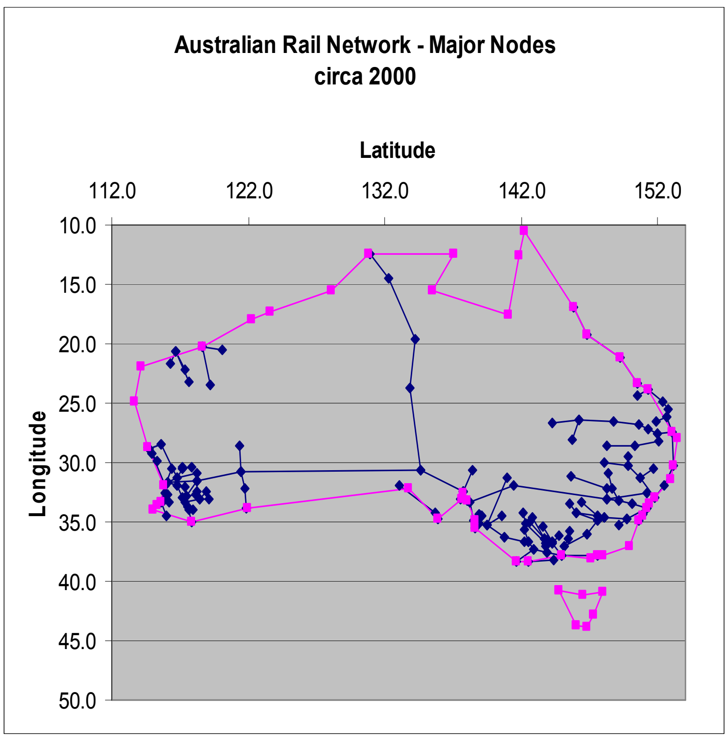

Figure 3: Australian Rail Network – Major Nodes

[4] www.railmaps.com.au

The graph of this network has 143 towns and 220 links G(143,220). Potentially there are 10,153 links if the network were totally interconnected so the existing connectivity matrix is quite sparse making manual computations feasible.

Figure 3 shows quite clearly that the rail networks have strong hubs at the capital cities since that is where the majority of the rail networks originated. Western Australia, South Australia and Victoria are strongly connected to the capital cities. New South Wales and Queensland have coastal ports that rail connects to inland mines. This is also the case in North Western Australia with the private iron ore rail lines.

An analysis of the nodal connectivity pattern is given in figure 4. This demonstrates quite effectively the scale free nature of the rail network. The two hub nodes with 5 other nodal connections are Melbourne and Adelaide. The nodes with 4 connections are in Western Australia, Kalgoorlie, Merredin and Perth and then Port Augusta South Australia and Sydney, New South Wales.

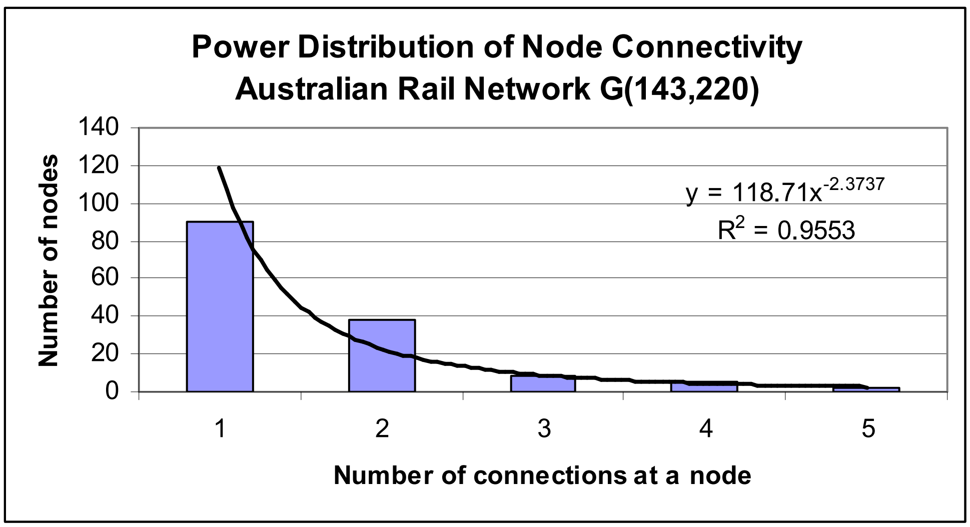

Figure 4: Nodal Connectivity of Australian Rail Network

The power distribution equation fits the data quite well ($r^2 = .95$) with $\gamma = 2.3$ which accords quite well with the literature on other scale free networks.

One of the characteristics of an optimal rail network should contain the criteria of distance minimisation since rail track is one of the most expensive items of any transport infrastructure. Track where possible is laid directly between two nodes or terminals subject to the grade of the land. A minimal spanning tree of the rail network would have the property that it would minimise track distance. Figure 5 shows the results of an Excel spreadsheet application of Prim's algorithm[5] applied to a 143 X 143 inter-nodal distance matrix where the nodes are given in Figure 3.

[5] Berry et.al. (1990) "Decision Mathematics", provide a simple example pages 182 – 186.

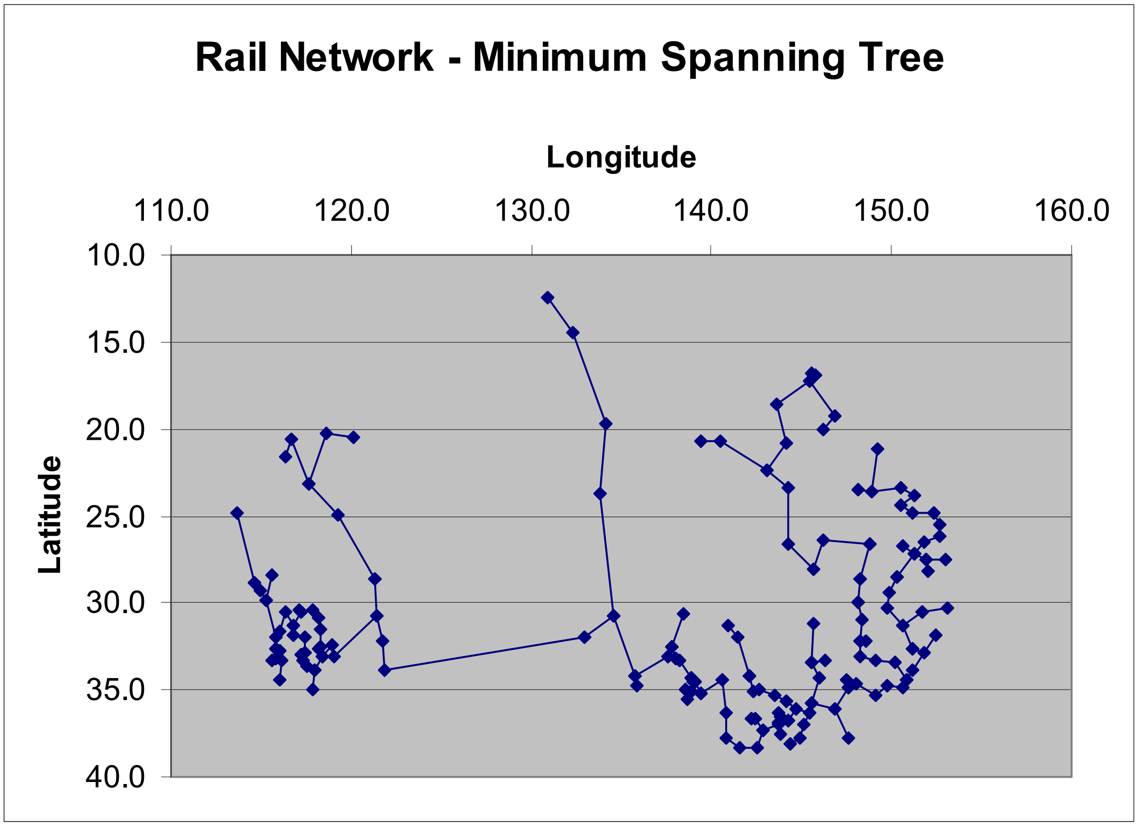

Figure 5: Minimal Spanning Tree of Australian Rail Network

The link or route distance of the minimal spanning tree is 18,107 kilometres compared with a distance of 22,606 kilometres for the sketch network as given by Figure 3. A comparison of the two maps shows that the largest connecting nodes have degree 3 and are Dalby in Queensland, Tocumwal in Victoria and Wollongong in New South Wales. The nodal focus on the capital cities and ports in Queensland and Western Australia is diminished. Instead, the minimal spanning tree develops a linear network stretching from Wollongong to Mt. Isa in Queensland. The minimal spanning tree has less “leaf like” network structures that are evident in the existing networks in the wheat belts of Western Australia and Victoria.

Figure 6 shows the power distribution of the nodes in the minimal spanning tree. Whilst not exhibiting as strong a power relationship as Figure 4, the results still show that there is a scale free distribution for the minimal spanning tree. However, the strength of the hubs has been substantially diminished – the network has been “flattened” somewhat.

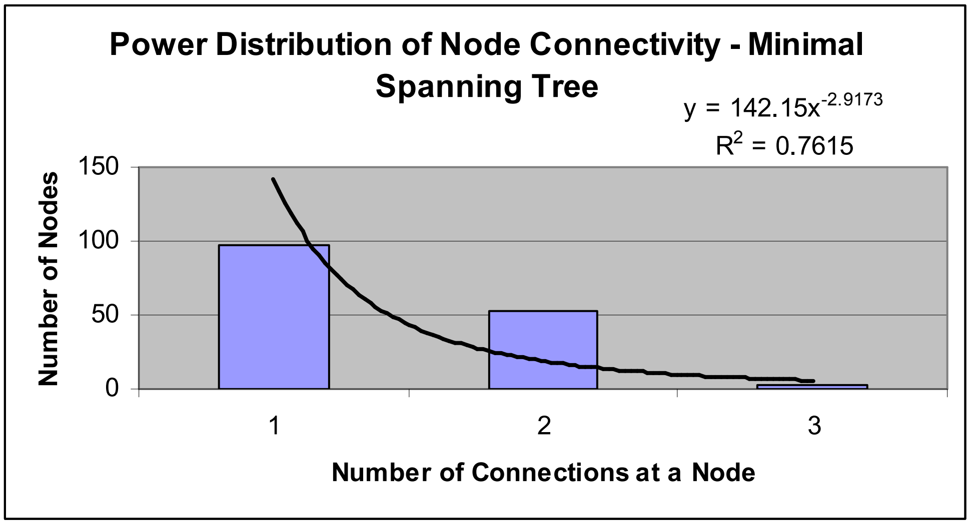

Figure 6: Power Distribution of Nodes of Minimal Spanning Tree.

Steiner "Type" Trees

A generalization of a minimal spanning tree is a Steiner tree[6]. Minimal spanning trees assume that nodes are fixed and limited but Steiner showed that by the inclusion of some new nodes a shorter minimal spanning tree could be created. It is a case of "thinking outside the square" or not being confined to the existing set of terminals or nodes. The classic example that is usually given is of 4 nodes that require connection (Figure 6).

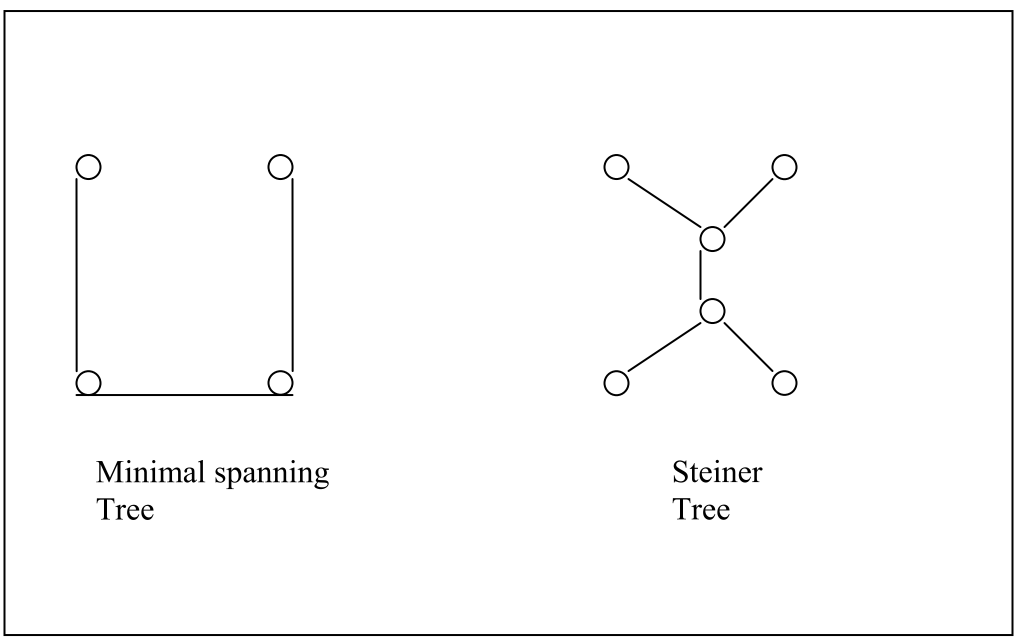

Figure 6: Minimal Spanning Tree and Steiner Tree

[6] Haggett and Chorley (1969) "Network Analysis in Geography", see pages 212-216.

Steiner trees are notoriously difficult to solve and are considered in a league with combinatorially hard problems such as Travelling Salesman. Nevertheless, it is possible to take the concept of additional nodes and design networks based about this concept.

A centre of gravity concept can be used to locate the centre of gravity of the rail terminals alone (the nodes). Figure 7 shows the location of the rail nodes without the links and the location of six hub locations which minimize the distance from the terminals. The analysis proceeded by first finding the centre of gravity of a single hub, then increasing the number of hubs incrementally. Excel's non-linear optimization routine was used to find the centres. The equation is straightforward:

$$\text{Minimise} \quad \sum \sum (Tij - Xij^{\,n})^{2}$$

where Tij are the terminals (220 nodes) and Xij are the locations of the n= 6 hubs.

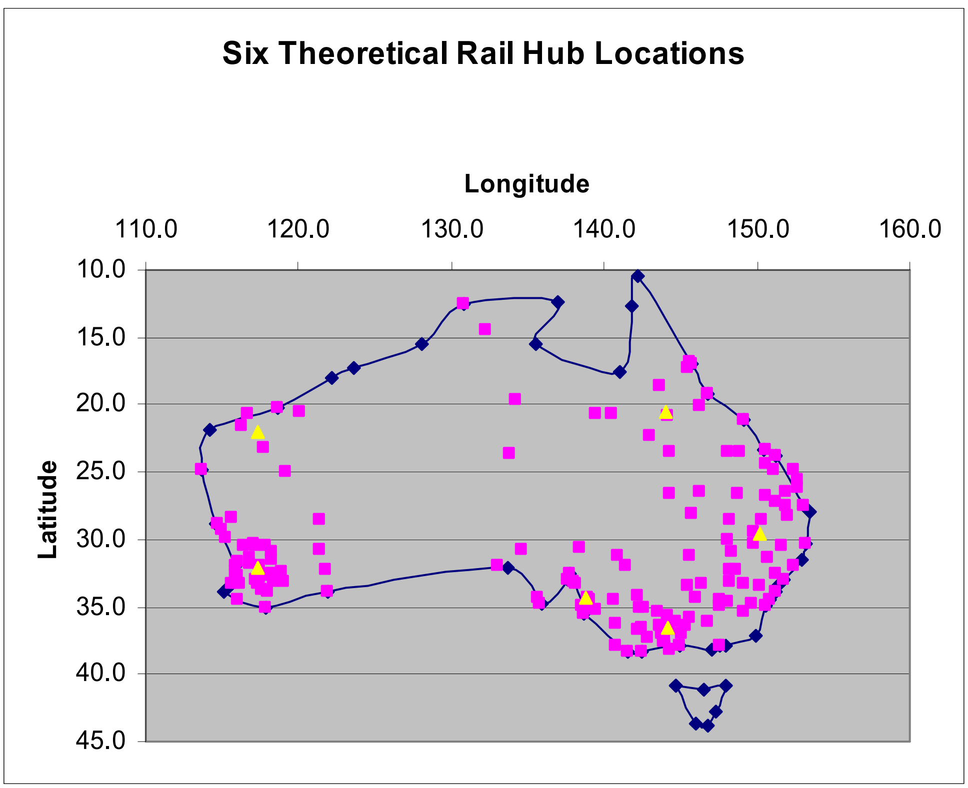

Figure 7: Terminal Rail Network Locations with Hub Locations.

A plot of the generalized transport cost of incrementally adding hubs is given in figure 8. This indicates that substantial costs can be achieved by adding hubs to the terminals. Six hubs were chosen because cost savings began leveling off after this number.

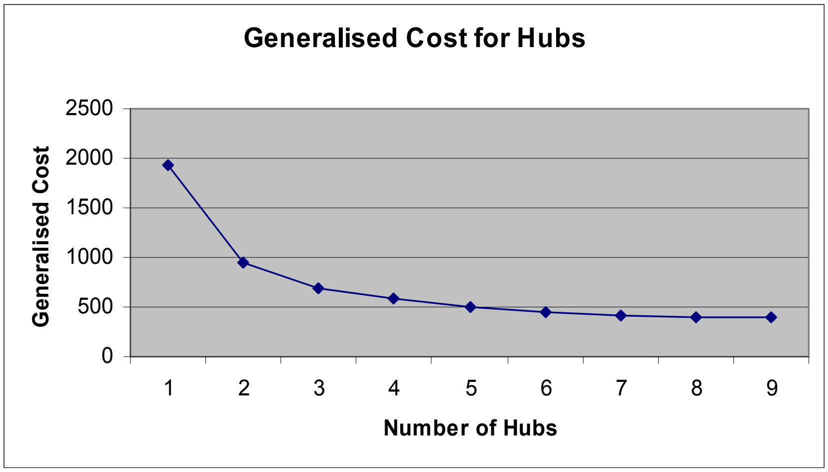

Figure 8: Incremental Costs for Hubs

The six hubs might be considered a rationalisation of the terminals. A minimal spanning tree connecting these hubs is given in Figure 9 along with a Steiner type tree in Figure 10. The minimal spanning tree algorithm was used for this simple six node network and a non-linear program with an objective function similar to that used for centre of gravity was used for to provide a Steiner type tree[7].

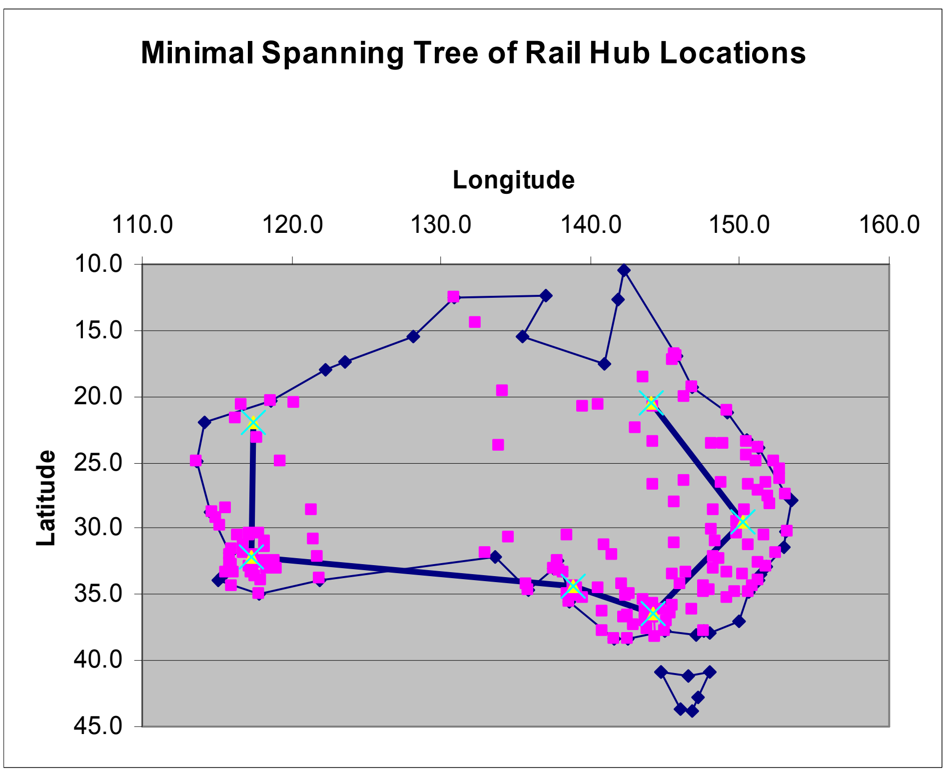

Figure 9: Minimal Spanning Tree for Six Rail Hubs.

[7] The heuristic used was not Dreyfus/Wagner which is a recognised solution procedure.

The work to-date on modeling a Steiner type tree suggests that it is marginally more expensive than the minimal spanning tree but this may be due to the inexact nature of the non-linear optimization approach utilized.

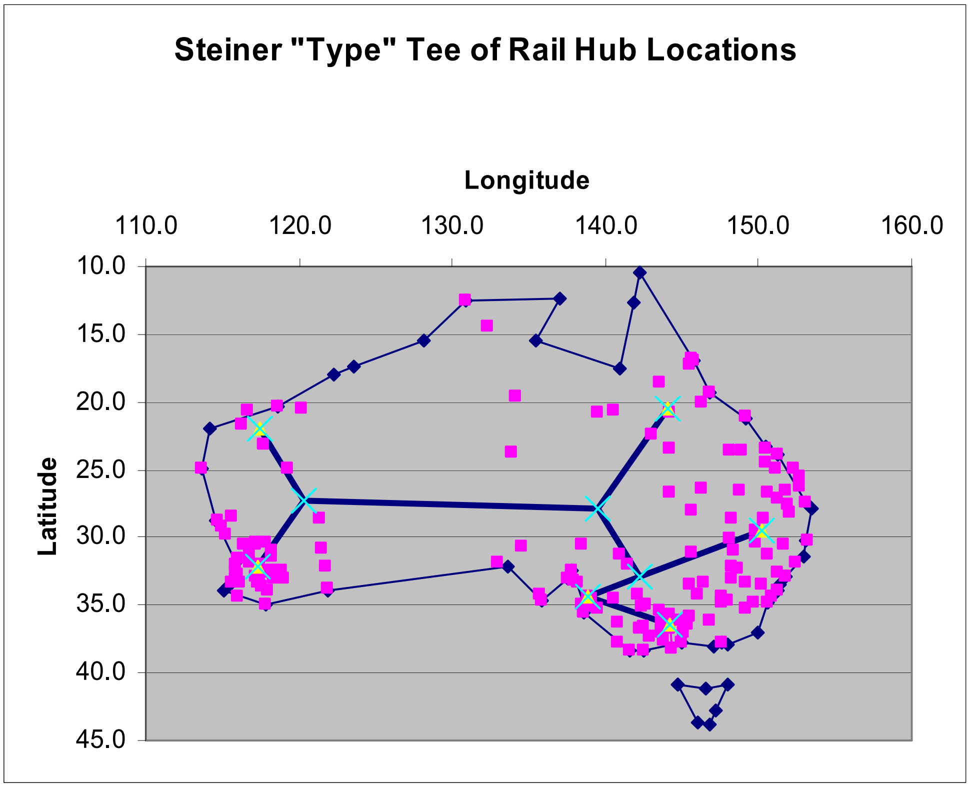

Figure 10: Steiner type tree for Six Hubs.

Conclusions

The analysis using the minimal spanning tree shows quite clearly the potential benefits from rethinking the structure of the rail network. The existing structure has evolved incrementally over the past one hundred and fifty years albeit that most of the rail network was in place by 1930. But little has been done to make the network more efficient. To cut costs pruning has been done of some of the least used leaves of the network and so there has been a reduction in track kilometers until quite recently. This is due of course to the Darwin to Alice Springs connection.

However, it suggests that planners should consider developing inland corridors. On the east coast, an inland north south corridor stretching from Orange through Dubbo and Cunnamulla to Mount Isa should be considered. Interestingly such a rail corridor was suggested in the book “The Line Ahead” Bright (1996). Similarly in the North West, connecting the iron ore rail lines with the Southern network in Western Australia is also worth further discussion and analysis. The major North South rail corridor from Adelaide to Darwin that has recently been completed suggests that the minimal spanning tree is a guideline for the future evolution of the rail network.

The Steiner type tree is significant to consider because it suggests that there may be great value in the creation of a set of hub terminals that minimize distances from the rail terminals of the network. It demonstrates the importance of rethinking the alignment of the main lines because their may be significant capital and operating cost savings if the network were redesigned from a global optimization perspective. The alignments were created essentially in the footsteps of the explorers and it is time to rethink the effectiveness of their original tracks.

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